

Exercise 8

Find the particular solution for each of the following initial value problems:

$$xu' + u = 2x, \quad u(1) = 1$$

Solution

Observe that the left side can be written as $(xu)'$ by the product rule.

$$\frac{d}{dx}(xu) = 2x$$

Now integrate both sides with respect to x .

$$xu = x^2 + C$$

The general solution is thus

$$u(x) = x + \frac{C}{x}.$$

Because an initial condition is given, this constant of integration can be determined.

$$u(1) = 1 + \frac{C}{1} = 1 + C = 1 \quad \rightarrow \quad C = 0$$

Therefore,

$$u(x) = x.$$